Modeling and Analysis of Collaborative Consumption in Peer-to-Peer Car Sharing

Saif Benjaafar[1], Guangwen Kong[1], Xiang Li[1], and Costas Courcoubetis[2]

[1]*Department of Industrial and Systems Engineering, University of Minnesota*

[2]*Engineering and Systems Design, Singapore University of Technology and Design*

1. INTRODUCTION

We are witnessing a paradigm shift away from the exclusive ownership and consumption of resources to one of shared use and consumption. This paradigm shift is taking advantage of innovative new ways of peer-to-peer sharing that are voluntary and enabled by internet-based exchange markets and mediation platforms. Value is derived from the fact that most resources are acquired to satisfy peak demand but are otherwise poorly utilized (e.g., the average car in the US is used less than 10 percent). Several successful businesses in the US and elsewhere, such as AirBnB for rooms in private homes, Uber for taxi service, LiquidOffice for office space, RelayRides for private car sharing, and TaskRabbit for errands, among many others, provide a proof of concept and evidence for the viability of the collaborative consumption concept. Collectively, these businesses and other manifestations of collaborative consumption are giving rise to what is becoming known as the sharing economy.

Collaborative consumption has the potential of increasing access while reducing investments in resources and infrastructure. In turn this could have the twin benefit of improving consumer welfare (individuals who may not otherwise afford a product now have an opportunity to use it) while reducing societal costs (externalities, such as pollution that may be associated with production, distribution use, and disposal of the product). Take cars for example. The availability of a sharing option is likely to lead some to forego car ownership in favor of on-demand access. In turn, this could result in a corresponding reduction in road capacity and parking infrastructure. However, increased collaborative consumption may have other consequences, some of which may be undesirable. For example, greater access to cars could increase car usage and, therefore, lead to more congestion and pollution if it is not accompanied by a sufficient reduction in the numbers of cars. This could occur if sharing leads to speculative investments in cars and price inflation, or if yet it affects the availability and pricing of other modes of public transport (e.g., taxis, buses, and trains).

Collaborative consumption raises several important questions. How does collaborative consumption affect ownership and usage of resources? Is it necessarily the case that collaborative consumption leads to lower ownership, lower usage, or both (and therefore to improved sustainability)? If not, what conditions would favor lower ownership, lower usage, or both? How would the platform set prices, commissions, and membership fees and under what conditions would choices for these parameters lead to socially desirable outcomes? To what extent would a private platform (a platform that maximizes its own profit) improve social welfare? How far would the resulting social welfare be from that obtained under a public platform (a platform that maximizes social welfare)? What public policies, if enacted, would ensure that collaborative consumption would lead to higher social welfare? In this paper and the extended version [1], we address these and other related questions in the context of peer-to-peer car sharing.

2. A BRIEF LITERATURE REVIEW

Our work is related to the literature on two-sided markets (see for example [5] and [4]) and network externalities ([3] and [2]). Examples of two-sided markets include video game platforms which need to attract both game developers to design games and game players to use the video game platform; social media which bring together members and advertisers; and operating systems for computers and smart phones, which connect users and application developers. A common feature of two-sided markets is that the utility of individuals on each side of the market increases with the size of the other side of the market. As a result, it can be beneficial for the platform to heavily subsidize one side of the market (e.g., facebook is free to subscribers). Collaborative consumption is different from two-sided markets in several ways, the most important of which is that the two sides are not distinct. In collaborative consumption, being either an owner or a renter is a decision that users of the platform make, with more owners implying fewer renters (and vice-versa). Therefore, heavily subsidizing one side of the market may not necessarily be desirable as it can create an unbalance in the supply and demand for the shared resource. Similarly, network externalities are not always positive (e.g., having more renters increases the demand for resources but reduces the number of these resources). Other related literature include literature on social sharing of information goods, secondary markets for durable goods, and on-demand mobility systems. Discussion of this literature can be found in [1].

3. MODEL DESCRIPTION

In this section, we describe a basic model of collaborative consumption. We focus on the case of car sharing. However, the model has potentially broad applicability to the collaborative consumption of other resources. We consider a population of individuals. In the absence of collaborative consumption, each individual makes a decision about whether or not to own a car (or more generally a resource). We assume cars (resources) are homogeneous in their features, quality, and cost of ownership so that uniform pricing is plausible (this is consistent with observed practices by certain peer-to-peer platforms such as Uber). In the presence of collaborative consumption, each individual decides on whether to own, rent from others who own, or neither. Owners incur the fixed cost of ownership but can now generate income by renting their cars to others who choose not to own. Renters pay the rental fee but avoid the fixed cost of ownership.

We let p denote the rental price per unit of usage (e.g., unit of time) that renters pay. This rental price may be set by a third party platform (an entity that may be motivated by profit, total social welfare, or some other concern) or it may arise naturally over time as part of an equilibrium (we focus for now on the former). The platform extracts a commission from successful transactions, which we denote by γ, where $0 \leq \gamma \leq 1$, so that the rental income seen by the owner per unit of usage is $(1-\gamma)p$. We let α, where $0 \leq \alpha \leq 1$ denote the probability in equilibrium that an owner, whenever she decides to put her car up for rent, is successful in finding a renter. Similarly, we denote by β, where $0 \leq \beta \leq 1$, the probability that a renter, whenever he decides to rent, is successful in finding an available car. The owner incurs a fixed cost of ownership, denoted by c. Whenever the car is rented, the owner incurs an additional cost, denoted by w, due to extra wear and tear the renter places on the car (a moral hazard the owner faces because of the renter's potential negligence and mishandling of the car). Renters, on the other hand, incur an inconvenience cost, denoted by d (in addition to paying the rental fee), from driving someone else's and not their own. Without loss of generality, we assume that $c, d, p, w \in [0,1]$. To allow for collaborative consumption to take place, we also assume that $(1-\gamma)p \geq w$.

Individuals are heterogeneous in the utility they derive from car usage, with their *type* characterized by their usage level ξ. We assume usage is exogenously determined and unaffected by the presence of collaborative consumption (i.e., the usage of each individual is mostly inflexible and must be satisfied through either renting or owning.). There may of course be settings where usage is, at least partially, discretionary and is jointly determined with the decision about whether to own, rent or neither. The utility derived by an individual with type ξ is denoted by $u(\xi)$. Without loss of generality, we normalize the usage level to $[0,1]$, where $\xi = 0$ corresponds to no usage at all and $\xi = 1$ to full usage. We let $F(\xi)$ denote the distribution function in the population and $f(\xi)$ the corresponding density function, where both are continuous functions.

The surplus of an owner with usage level ξ can now be expressed as

$$\pi_o(\xi) = u(\xi) + (1-\xi)\alpha[(1-\gamma)p - w] - c, \qquad (1)$$

while the surplus of a renter as

$$\pi_r(\xi) = u(\xi\beta) - (p+d)\xi\beta. \qquad (2)$$

An individual with type ξ would participate in collaborative consumption as an *owner* if conditions $\pi_o(\xi) \geq \pi_r(\xi)$ and $\pi_o(\xi) \geq 0$ are satisfied. The first constraint is an incentive compatibility constraint that ensures that an individual with type ξ prefers to be an owner rather than a renter. The second constraint is a participation constraint that ensures the individual participates in collaborative consumption[1]. Similarly, an individual with type ξ would participate in collaborative consumption as a *renter* if conditions $\pi_r(\xi) \geq \pi_o(\xi)$ and $\pi_r(\xi) \geq 0$ are satisfied.

If $\pi_o(\xi) - \pi_r(\xi)$ is monotonically increasing in ξ and $u(\theta) \geq (p+d)\theta$ for $\theta \in [0,1]$, collaborative consumption would take place if there exists $\theta \in (0,1)$ such that

$$\pi_o(\theta) = \pi_r(\theta). \qquad (3)$$

The parameter θ would then segment the population into owners and renters, where individuals with $\xi \geq \theta$ are owners and individuals with $\xi < \theta$ are renters. We refer to $\omega = 1-\theta$, the fraction of owners in the population, as the *ownership* level.

In the absence of collaborative consumption, an individual would own a car if $u(\xi) \geq c$ and would not otherwise. Let $\hat{\theta}$ denote the solution to $u(\xi) = c$. Then, the fraction of the population that corresponds to owners (ownership) is $\hat{\omega} = 1 - \hat{\theta}$ and to non-owners is $\hat{\theta}$.

3.1 Matching Supply with Demand

In the presence of collaborative consumption, let $D(\theta)$ denote the aggregate demand (for car rentals) generated by renters and $S(\theta)$ the aggregate supply generated by owners, for given θ. Then,

$$D(\theta) = \int_{[0,\theta)} \xi f(\xi) d\xi \text{ and } S(\theta) = \int_{[\theta,1]} (1-\xi) f(\xi) d\xi.$$

In addition, we let $q(\theta)$ denote the total usage (the sum of usage by the owners and the renters). Then, $q(\theta) = \int_{[\theta,1]} \xi f(\xi) d\xi + \beta \int_{[0,\theta]} \xi f(\xi) d\xi$, where the first term is usage due to owners and the second term is usage due to renters.

For a given θ, the amount of demand from renters that is fulfilled must equal the amount of supply from owners that is matched with renters. In other words, for a given θ, the following fundamental relationship must be satisfied

$$\alpha S(\theta) = \beta D(\theta). \qquad (4)$$

The parameters α and β, along with θ, are determined endogenously in equilibrium.

Let $\rho(\theta) = \frac{D(\theta)}{S(\theta)}$. Then $\rho(\theta)$ can be viewed as a measure of the relative demand for the available cars. A higher $\rho(\theta)$ indicates that it is more likely for an owner to rent her car, implying a higher owner surplus. However, a higher $\rho(\theta)$ also indicates that a renter is less likely to find an available car, implying a lower renter surplus. Hence, with collaborative consumption, there is ongoing tension between having too many renters and too many owners. This tension is resolved in equilibrium via θ, which balances the surplus of owners and renters and determines the fraction of each in the population.

A model for α and β is one that arises naturally from a *multi-server loss queueing system*. In the corresponding queueing system, the arrival process is that of rental requests. If we let m denote the mean rental time per each rental, the arrival rate (in terms of rental request per unit time) is given by $\lambda(\theta) = D(\theta)/m$. The service capacity in the system (the number of rental requests that can be fulfilled per unit time) is given by $C(\theta) = \frac{S(\theta)}{m}$, where $S(\theta)$

[1]The value of the outside option (e.g., using public transport) is normalized, without loss of generality, to zero.

is the number of available cars for a given θ. Noting that $C(\theta)$ also satisfies $C(\theta) = S(\theta)\mu$, where μ is the average number of rentals that can be handled by each car per unit time, we recover the familiar expression of $\rho(\theta) = \frac{\lambda(\theta)}{S(\theta)\mu}$. In such a system, $1 - \beta$ would correspond to the blocking probability (the probability that a request for rental finds all cars rented out, or, in queueing parlance, a request finds all servers busy) and α corresponds to the probability that an available car (server) is rented (busy). Assuming the arrival of rental requests can be approximated by a Poisson process, we can approximate α as follows (see for example [6])

$$\alpha = \frac{\rho(\theta)}{1 + \rho(\theta)}. \tag{5}$$

Applying Little's law leads to $\beta = \frac{1}{1+\rho(\theta)}$. As a result, we have $\alpha + \beta = 1$. Note that α and β, as defined, satisfy the balance equation (4). An equilibrium under collaborative consumption exists if there exists $(\theta, \alpha) \in (0, 1)^2$ that is solution to Equations (3) and (5). When it exists, we denote this solution by (θ^*, α^*). Knowing the equilibrium allows us to answer important questions regarding car ownership, overall usage, and social welfare, among others.

3.2 The Platform's Problem

We have so far treated the rental fee p and the commission γ as exogenous parameters. However, they may be decided upon by the platform (as we discuss later, the platform may also decide on fixed membership fees). Platforms may have different objectives. For example, a privately owned platform may be interested in maximizing the revenue generated from successful transactions. In contrast, a government owned platform may be interested in maximizing total social welfare, while a not for profit owned platform may be interested in minimizing an externality such as pollution from emissions.

For a revenue-maximizing platform, the optimization problem can be stated as follows.

$$\max_{p,\gamma} v_r(p) = \gamma p \alpha S(\theta), \tag{6}$$

subject to constraints (3) and (5).

For a social welfare-maximizing platform, the objective is to maximize the sum of owners and renters' surpluses. Thus, the platforms problem can be stated as

$$\max_{p,\gamma} v_s(p) = \int_{[\theta,1]} (u(\xi) - (1-\xi)\alpha w - c) f(\xi) d\xi + \int_{[0,\theta]} (u(\xi\beta) - d\xi\beta) f(\xi) d\xi, \tag{7}$$

subject to constraints (3) and (5).

4. EQUILIBRIUM ANALYSIS

In this section, we focus on the case where the utility function has the linear form $u(\xi) = \xi^2$, and ξ is uniformly distributed in $[0, 1]$. We do so for ease of exposition and to allow for closed form expressions. In this case, we must have $(1 - \gamma)p \geq w$ and $p \leq 1 - d$ (Otherwise, collaborative consumption will not take place.). We denote the set of admissible prices by

$$P(\gamma, w, d) = \{p | (1 - \gamma)p \geq w, p \leq 1 - d\}. \tag{8}$$

Letting θ denote the solution to $\pi_o(\xi) = \pi_r(\xi)$ leads to

$$\theta = \frac{c - ((1-\gamma)p - w)\alpha}{p + d + (1 - p - d)\alpha - ((1-\gamma)p - w)\alpha}. \tag{9}$$

Given θ, the aggregate demand under collaborative consumption is given by $D(\theta) = \frac{\theta^2}{2}$ and aggregate supply by $S(\theta) = \frac{(1-\theta)^2}{2}$. This leads to $\rho(\theta) = \frac{\theta^2}{(1-\theta)^2}$, and by (5)

$$\alpha = \frac{\theta^2}{(1-\theta)^2 + \theta^2}. \tag{10}$$

An equilibrium exists if equations (9) and (10) admit a solution (θ^*, α^*) such that $(\theta^*, \alpha^*) \in (0, 1)^2$. In the following theorem, we establish the existence and uniqueness of such an equilibrium. Let

$$\Omega = \{(p, \gamma, c, w, d) | c \in (0, 1), (\gamma, w, d) \in [0, 1]^3, p \in P(\gamma, w, d)\}.$$

THEOREM 1. *A unique equilibrium (θ^*, α^*) exists for each $(p, \gamma, c, w, d) \in \Omega$.*

The following lemma describes how the equilibrium (θ^*, α^*) varies with the price p, commission γ, cost of ownership c, wear and tear cost w and inconvenience cost d.

LEMMA 2. *$(\theta^*, \alpha^*) : \Omega \to (0, 1)^2$ is continuous on Ω, and continuously differentiable on Ω°. Moreover, $\frac{\partial \theta^*}{\partial p} < 0$, $\frac{\partial \alpha^*}{\partial p} < 0$, $\frac{\partial \theta^*}{\partial \gamma} > 0$, $\frac{\partial \alpha^*}{\partial \gamma} > 0$, $\frac{\partial \theta^*}{\partial c} > 0$, $\frac{\partial \alpha^*}{\partial c} > 0$, $\frac{\partial \theta^*}{\partial w} > 0$, $\frac{\partial \alpha^*}{\partial w} > 0$, $\frac{\partial \theta^*}{\partial d} < 0$, and $\frac{\partial \alpha^*}{\partial d} < 0$.*

Lemma 2 indicates that, in equilibrium, the population of renters θ^* increases with the commission γ, cost of ownership c and wear and tear cost w, but decreases with rental price p and inconvenience cost d. In addition, in equilibrium, the probability that a car owner is successful in renting her car α^* increases with γ, c and w, and decreases with p and d. These results are consistent with intuition.

In the presence of collaborative consumption, ownership in equilibrium, which we denote by ω^*, and total usage level, which we denote by q^*, are respectively given by $\omega^* = 1 - \theta^*$ and $q^* = \frac{1-\alpha^* \theta^{*2}}{2}$.

PROPOSITION 3. *(i) $\frac{\partial \omega^*}{\partial p} > 0$, $\frac{\partial \omega^*}{\partial \gamma} < 0$, $\frac{\partial \omega^*}{\partial c} < 0$, $\frac{\partial \omega^*}{\partial w} < 0$ and $\frac{\partial \omega^*}{\partial d} > 0$; (ii) $\frac{\partial q^*}{\partial p} > 0$, $\frac{\partial q^*}{\partial \gamma} < 0$, $\frac{\partial q^*}{\partial c} < 0$, $\frac{\partial q^*}{\partial w} < 0$ and $\frac{\partial q^*}{\partial d} > 0$.*

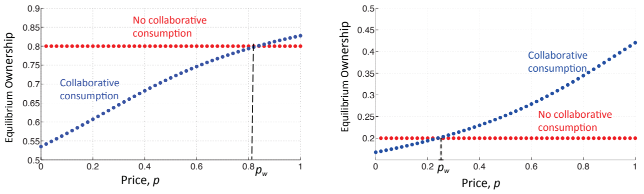
Proposition 3 is a direct consequence of Lemma 2. It shows that, in equilibrium, ownership decreases with the commission γ, cost of ownership c and wear and tear cost w, but increases with price p and inconvenience cost d. Similarly, usage increases with the rental price and the inconvenience cost, and decreases with the commission fee, ownership cost and wear and tear cost.

While these monotonicity results are perhaps expected, it is not clear how ownership and usage under collaborative consumption compare to those under no collaborative consumption. In what follows, we provide comparisons between systems with and without collaborative consumption,

[2] The utility function has constant returns to scale, and the utility derived from each unit of usage is normalized to 1. For there to be renters, we must have $p \in [0, 1]$. Similarly, we may assume without loss of generality that $c, w, d \in [0, 1]$. It is also straightforward to consider a utility function in general linear form and carry out similar analysis.

and address the questions of whether or not collaborative consumption reduces car ownership and usage.

In the absence of collaborative consumption, ownership and usage, denoted respectively by $\hat{\omega}$ and $\hat{q}$, are given by $\hat{\omega} = 1 - c$ and $\hat{q} = \frac{1-c^2}{2}$.

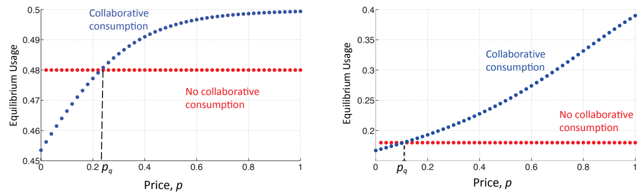


(a) $c = 0.2$, $\gamma = 0.2$ (b) $c = 0.8$, $\gamma = 0.2$

Figure 1: Impact of Price on Ownership

Proposition 4. *Suppose $\gamma \neq 1$, $w < (1-\gamma)(1-d)$ and $p_\omega = \frac{(1-d)(1-c)+wc}{1-\gamma c}$. Then, $p_\omega \in P(\gamma, w, d)^\circ$, $\omega^* < \hat{\omega}$ if $p < p_\omega$, $\omega^* > \hat{\omega}$ if $p > p_\omega$, and $\omega^* = \hat{\omega}$ if $p = p_\omega$. Moreover, $\frac{\partial p_\omega}{\partial \gamma} > 0$, $\frac{\partial p_\omega}{\partial c} < 0$, $\frac{\partial p_\omega}{\partial w} > 0$, and $\frac{\partial p_\omega}{\partial d} < 0$.*

The result above shows that depending on the rental price p, collaborative consumption can result in either lower or higher ownership. In particular, when the rental price p is sufficiently high (above the threshold p_ω), collaborative consumption leads to higher ownership (more cars). Moreover, the threshold above which prices must be for this to occur is decreasing in the cost of ownership and renter's inconvenience, and increasing in the commission fee and wear and tear cost. This is perhaps surprising as it shows that collaborative consumption is more likely to lead to more cars (and not less) when the cost of owning a car is high. Collaborative assumption in this case allows individuals to offset the high ownership cost and pulls in a segment of the population that may not otherwise choose to own. The reverse is of course also true. Collaborative consumption is more likely to lead to lower car ownership when the ownership cost is low. These effects are illustrated for an example system in Figure 1.



(a) $c = 0.2$, $\gamma = 0.2$ (b) $c = 0.8$, $\gamma = 0.2$

Figure 2: Impact of Price on Usage

Similarly, usage can be either lower or higher with collaborative consumption than without it. In this case, there is again a price threshold above which usage is higher with collaborative consumption, and below which usage is higher without collaborative consumption. When either w or d is sufficiently high, collaborative consumption always leads to higher usage. The result is formally stated in the proposition below and illustrated in Figure 2.

Proposition 5. *One of the following is true:*
(i) There exists $p_q \in P(\gamma, w, d)^\circ$ such that $q^* < \hat{q}$ if $p < p_q$, $q^* > \hat{q}$ if $p > p_q$, and $q^* = \hat{q}$ if $p = p_q$, or
(ii) $q^* \geq \hat{q}$ for all $p \in P(\gamma, w, d)$.
In fact, (i) is true iff w and d are both sufficiently low, especially when w and d are both 0. Moreover, there exists $w_q \geq 0$ such that (i) is true if $w < w_q$ and (ii) is true otherwise, with w_q decreasing in d. Similarly, there exists $d_q \geq 0$ such that (i) is true if $d < d_q$ and (ii) is true otherwise, with d_q decreasing in w.

These results could have implications for public policy. In regions, where the cost of car ownership is high, the results imply that, unless rental prices are kept sufficiently low or the commission extracted by the platform is made sufficiently high, collaborative consumption would lead to more cars and more usage not less. On the other hand, in regions where both cost of ownership and rental prices are low, it may be desirable to encourage collaborative consumption as it can have the double benefit of reducing both the number of cars and the amount of usage. This is illustrated in Figure 3.

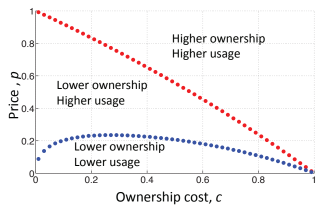


Figure 3: Ownership and Usage for Varying Rental Price and Ownership Cost

5. CONCLUDING COMMENTS

In the extended version of the paper [1], we study car sharing under both revenue-maximizing and welfare-maximizing platforms. We examine how the platforms would maximize their objectives with respect to price and commission. We show that in equilibrium collaborative consumption could still lead to higher ownership and higher usage. We also compare the resulting social welfare between the two systems.

6. REFERENCES

[1] S. Benjaafar, G. Kong, X. Li, and C. Courcoubetis. Modeling and analysis of collaborative consumption in peer-to-peer product sharing. *Working paper*, 2015.

[2] M. L. Katz and C. Shapiro. Network externalities, competition, and compatibility. *RAND Journal of Economics*, 34(2):247–265, 2003.

[3] S. J. Liebowitz and S. E. Margolis. Network externality: An uncommon tragedy. *Journal of Economic Perspectives*, 8(2):133–150, 1994.

[4] S. Mongell and A. E. Roth. Sorority rush as a two-sided matching mechanism. *The American Economic Review*, 81(3s):441–464, 1991.

[5] J.-C. Rochet and J. Tirole. Two-sided markets: A progress report. *The RAND Journal of Economics*, 37(3):645–667, 2006.

[6] M. J. Sobel. Note-simple inequalities for multiserver queues. *Management Science*, 26(9):951–956, 1980.