1. INTRODUCTION

The U.S. department of energy recently listed “development of rules for market evolution that enable system flexibility” among the key strategic areas of intervention for a successful integration of renewable energy into the grid [2]. Renewable energy generation from wind and solar is intermittent and its prediction accuracy is precise only within a short time horizon (e.g., 5-15 minutes [1]). Therefore, new market mechanism solutions such as intra-day markets are proposed that allow for flexible generation of renewable energy to exploit the improved forecast accuracy of renewable resources over time [14]. In this paper, we study sequential contract design problems that incorporate the arrival of new information about renewable generation and allow for flexible production.

Today, the renewable energy generation receives extra-market treatment such as feed-in tariffs, guaranteed grid access, and lenient penalty rate [3, 7]. For example, the Participating Intermittent Resource Program (PIRP) mandates the California independent system operator to accept all the wind generation in real-time and treat them as negative loads. The subsequent increased cost of the required reserve generation capacity is then socialized among the load serving entities (LSE). However, such approaches cannot be sustained for high levels of renewable generation as the imposed reserve generation cost on LSEs becomes excessively high and results in social welfare loss. In the long-run, renewable energy generation needs to participate in electricity markets and be exposed to market mechanisms.

An alternative approach, implemented in the U.K., requires the wind generators to bid in conventional electricity markets and pay penalty for ex-post deviation from their ex-ante contracted schedule. Such a firm contracting approach is the subject of many studies in the literature. The works of [4, 15] study the problem of optimal bidding in a two-settlement market structure with an exogenous price and penalty rate. The problem of mechanism design for wind aggregation among many wind producers that jointly participate in a two-settlement market structure with exogenous price and penalty rate is investigated in [8, 10]. The authors in [12] study the problem of auction design for such a two-settlement market structure.

In this paper, we propose a simple two-step model to capture the dynamic variable nature of renewable generation and provide a general formulation for flexible forward contracts design for renewable resources. We formulate and solve two sequential flexible contract design problems, assuming private and public observation of the ex-post information (e.g., wind speed realization), respectively. We compare the flexible sequential contracts with the corresponding firm contract. We show that all three forward contract schemes can be interpreted as a menu of payment vs. quantity curves offered by the buyer to the seller. The seller chooses ex-ante one curve based on his ex-ante information, and after he receives his ex-post information about the availability of renewable resources, chooses one point on the selected curve. We show that the above described firm contract scheme can be interpreted as a special case of the flexible sequential forward contracts presented in this paper. The inclusion of ex-post information in the design of forward contracts enriches the space of allocation functions and enables the buyer to further refine the allocation. We prove that sequential flexible contracts are more beneficial for the buyer than firm contracts. We also show that the monitoring of ex-post information, if possible (e.g., wind speed), benefits the buyer. However, the effect of such monitoring on social welfare is more subtle and depends on the environment. We would like to note that the idea of flexible (non-firm) energy delivery has been studied in the literature in the context of energy pricing by the seller. The authors in [11] study the problem of efficient pricing of interruptible power services. The authors in [5] study the problem of optimal pricing for deadline differentiated deferrable loads (see [13] for a more comprehensive literature review).

2. MODEL

There is one buyer (she) and one seller (he) who want to sign a forward contract at time \( t = 1 \) for an energy trade taking place at time \( t = 2 \). The buyer gets utility \( \sum(q) \) from receiving energy \( q \); \( \sum(q) \) is increasing and strictly concave in \( q \). The seller has production cost \( C(q, \theta) \) parametrized by the seller’s type \( \theta \). Define \( \nu(q) := \frac{\partial \sum(q)}{\partial q} \) and \( c(q, \theta) := \frac{\partial C(q, \theta)}{\partial q} \) as the buyer’s marginal utility and the seller’s marginal production cost, respectively. We assume that at \( t = 1 \), ex-ante the seller has imperfect private information \( \tau \) about \( \theta \). We call \( \tau \) the seller’s ex-ante type. We assume that \( \tau \) takes two values \( L \) and \( H \) with probability \( p_L \) and \( p_H \), respectively.

The seller receives more information \( \omega \in [\omega_L, \omega_H] \) about his cost over time and can refine his private information about \( \theta \) at \( t = 2 \) as \( \theta = \Theta(\tau, \omega) \). We call \( \theta \) and \( \omega \) the seller’s ex-post type and shock, respectively. The conditional probability distribution of \( \theta \) given \( \tau \) is denoted by \( F_{\tau}(\theta) \), and the corresponding probability density function by \( f_{\tau}(\theta) \).
Assumption 1. The distribution \( F_{\tau}(\theta) \) has non-shifting support, i.e. \( f_{\tau} \) takes value on the interval \([\underline{\theta}, \overline{\theta}]\) with positive probability for \( \tau = L, H \).

Assumption 1 on non-shifting support is a standard assumption in the sequential contract design literature (first appearing in [6]). We assume, without loss of generality (see [13]), that the distribution of \( \omega \) is independent of \( \tau \), and \( \Theta(\tau, \omega) \) is increasing in \( \omega \). Let \( G(\omega) \) denote the probability distribution of \( \omega \), and \( g(\omega) \) the corresponding probability density function. The seller’s ex-ante information \( \tau \) can reveal different types of partial information about his final production cost. The seller’s ex-ante information \( \tau \) makes to the seller. Then, the buyer’s utility from direct incentive compatible (IC). We assume that the buyer is the generation \( \tau \omega^3 \) at zero marginal cost, where \( \omega \) denotes the realized wind speed. If \( \lambda \) denotes the penalty rate for shortfalls, then his production cost can be written as \( C(q; \theta) = C(q; \Theta(\tau, \omega)) = \lambda \max \{ q - \tau \omega^3, 0 \} \), \( (1) \) which satisfies condition (i) of assumption 2.

Finally, we make the following technical assumption. Define \( c_{\omega}(q; \theta) := \frac{\partial c(q; \theta)}{\partial \omega} \).

Assumption 3. The term \( c(q; \theta) + \frac{p_{L}}{p_{L}} \left[ \frac{F_{L}(\theta) - F_{L}(\theta)}{f_{L}(\theta)} \right] c_{\omega}(q; \theta) \) is increasing in \( \theta \) for \( \forall \phi \geq 0, \theta \in [\underline{\theta}, \overline{\theta}] \).

Assumption 3 is similar to the standard increasing hazard rate assumption in the static contract design literature that guarantees the monotonicity of allocation. Note that for an arbitrary cost function \( C(q; \theta) \) and conditional distribution \( F_{\tau}(\theta), \tau = L, H \), that satisfy assumptions 1 and 2, assumption 3 is satisfied for small enough \( p H^3 \).

3. MECHANISM DESIGN PROBLEMS

We assume full commitment for the buyer and the seller, invoke the revelation principle for multistage games [9], and restrict attention to direct revelation mechanisms that are incentive compatible (IC). We assume that the buyer is the mechanism designer, and both the buyer and the seller have quasi-linear utility. Let \( t \) denote the payment that the buyer makes to the seller. Then, the buyer’s utility from direct mechanism \( q(\tau, \omega), t(\tau, \omega) \) is given by

\[
E_{r,\omega}[V(q(\tau, \omega)) - t(\tau, \omega)] = p_{L}(S_{L} - R_{L}) + p_{H}(S_{H} - R_{H}) = S - R, \tag{2}
\]

\( ^{1}\)Example 1 does not satisfy assumption 3 in its current form since \( \lambda \max \{ q - \tau \omega^3 \} \) is not differentiable. However, \( \max \{ q - \tau \omega^3, 0 \} \) can be replaced by a differentiable approximation of \( \lambda \max \{ q - \tau \omega^3, 0 \} \) that satisfies assumption 3.

where \( S_{\tau} := E_{F_{q}} \{ V(q(\tau, \omega)) - C(q; \Theta(\tau, \omega)) \} \) and \( R_{\tau} := E_{F_{q}} \{ t(\tau, \omega) - C(q; \Theta(\tau, \omega)) \} \) denote the social welfare from a seller with ex-ante type \( \tau \) and the seller’s information rent (utility) with ex-ante type \( \tau \), respectively. Accordingly, \( S := \sum p_{L} S_{L} \) and \( R := \sum p_{H} R_{H} \) denote the total social welfare and information rent, respectively. Let \( q^{*} \) denote the efficient allocation given by

\[
v(q^{*}(\tau, \theta), \theta) = c(q^{*}(\tau, \theta), \theta), \tag{3}\]

Since \( \Theta(\tau, \omega) \) is strictly increasing in \( \omega \), one can rewrite the functions \( q \) and \( t \) in terms of \( \tau \) and \( \theta \) as \( q(\tau, \Theta(\tau, \omega)) = q(\tau, \omega) \) and \( t(\tau, \Theta(\tau, \omega)) = t(\tau, \omega) \). In the following, we use both \( (q, \omega) \) and \( (\tilde{q}, \omega) \) interchangeably wherever the use of each one simplifies the exposition of the results.

To demonstrate the advantage of incorporating the uncertainty into the mechanism design problem and making the allocation function \( q \) dependent on the shock \( \omega \), we consider forward contract design for the following scenarios: firm forward contract, flexible contract with public shock, and flexible contract with private shock.

3.1 Firm Contract

In a firm forward contract scheme, the buyer and the seller agree upon a fixed allocation that does not depend on any information that they acquire afterwards, that is, \( q^{*}(\tau, \omega) = q^{*}(\tau) \) for all \( \omega \). We assume that the ex-ante type \( \tau \) is private. Then, by the revelation principle, the firm forward mechanism \( \{ q^{*}(\tau), t^{*}(\tau) \} \) should be incentive compatible with respect to the ex-ante type \( \tau \). We note that \( t^{*}(\tau) \) denotes the expected payment associated with allocation \( q^{*}(\tau) \), which may include the (random) penalty for shortfalls as in example 1, i.e. \( t^{*}(\tau) = E_{F_{q}}[t^{*}(\tau) - \lambda \max \{ q^{*}(\tau) - \tau \omega^3, 0 \}] \). Define

\[
U^{f}(\tau_{r}; \tau) := t^{*}(\tau_{r}) - E_{F_{q}} \left\{ C(q^{*}(\tau_{r}); \Theta(\tau, \omega)) \right\}, \tag{4}\]

as the seller’s expected revenue with ex-ante type \( \tau \) reporting \( \tau_{r} \). With some abuse of notation define \( U^{f}(\tau) := U^{f}(\tau_{r}; \tau) \). Then, incentive compatibility for ex-ante type \( \tau \) can be written as

\[
IC_{U^{f}}^{\tau_{r}} : U^{f}(\tau_{r}) \geq U^{f}(\tau'_{r}; \tau) \quad \forall \tau', \tau \in \{ L, H \}. \tag{5}\]

For an incentive compatible \( \{ q^{f}, t^{f} \} \) that satisfies (5), we have \( R^{f} = U^{f}(\tau) \). The interim individual rationality (IR) for the seller can be written as

\[
IR^{f} : U^{f}(\tau) \geq 0 \quad \forall \tau \in \{ L, H \}. \tag{6}\]

The optimal firm forward contract design with interim IR can then be written as

\[
\max \space S - R \quad \text{subject to (5) and (6).}
\]

Theorem 1. With two ex-ante types, the allocation \( q^{f}(\tau) \) of the optimal firm forward contract satisfies

\[
v(q^{f}(L)) = -E_{F_{q}} \left\{ E_{R}[c(q^{f}(L); \theta)] \right\} \frac{p_{L}}{p_{L}} \left[ E_{F_{q}}[c(q^{f}(L); \theta)] - E_{F_{H}}[c(q^{f}(L); \theta)] \right], \tag{7}\]

\[
v(q^{f}(H)) = E_{F_{q}} \left\{ c(q^{f}(H); \theta) \right\} \tag{8}\]

the associated payment function \( t^{f}(\tau) \) is given by
3.2 Flexible Contract with Public Shock

The buyer and the seller want to sign a contract that depends on both the ex-ante type \( \tau \) and the shock \( \omega \). Assume that ex-ante type \( \tau \) is the seller’s private information, but the shock \( \omega \) is observed by both the buyer and the seller. Then, by the revelation principle, the contract \( \{q^*(\tau, \omega), t^*(\tau, \omega)\} \) needs to be incentive compatible with respect to ex-ante type \( \tau \). Define

\[
U^e(\tau', \tau) := \mathbb{E}_\omega \left\{ t^e(\tau', \omega) - C(\theta(\tau', \omega); \Theta(\tau, \omega)) \right\},
\]

as the seller’s expected revenue with ex-ante type \( \tau' \) reporting \( \tau' \). With some abuse of notation define \( U^e(\tau) := U^e(\tau, \tau) \). Then, incentive compatibility for ex-ante type \( \tau \) can be written as

\[
IC^e_{\tau} : \quad U^e(\tau) \geq U^e(\tau', \tau) \quad \forall \tau', \tau \in \{L, H\}. \tag{11}
\]

Note that for an incentive compatible \( (q^*, t^*) \) that satisfies (11), we have \( R_\tau = U^e(\tau) \). The interim IR for the seller can be written as

\[
IR_\tau : \quad U^e(\tau) \geq 0 \quad \forall \tau \in \{L, H\}. \tag{12}
\]

The optimal forward contract design with public shock and interim IR can then be written as

\[
\max S - R \quad \text{subject to (11) and (12).}
\]

**Theorem 2.** With two ex-ante types, the allocation \( q^*(\tau, \omega) \) of the optimal flexible contract with public shock satisfies

\[
v(q^*(L, \omega) = c(q^*(L, \omega); \Theta(L, \omega)) + \frac{p_H}{p_L} \left[ c(q^*(L, \omega); \Theta(L, \omega)) - c(q^*(L, \omega); \Theta(H, \omega)) \right], \tag{13}
\]

\[
v(q^*(H, \omega)) = c(q^*(H, \omega); \Theta(H, \omega)) \tag{14}
\]

and the associated payment function \( t^*(\tau, \omega) \) is given by

\[
t^*(L, \omega) = C(q^*(L); \Theta(L, \omega)), \tag{15}
\]

\[
t^*(H, \omega) = C(q^*(H); \Theta(H, \omega)) + \left[ C(q^*(L); \Theta(L, \omega)) - C(q^*(L); \Theta(H, \omega)) \right]. \tag{16}
\]

Note that the allocation for the ex-ante type \( H \) becomes efficient with the inclusion of the ex-post information \( \omega \). The allocation for the ex-ante type \( L \) is distorted from the efficient allocation \( q^* \) to reduce the information rent for the ex-ante type \( H \). The IR constraint is binding for the ex-ante type \( L \). The ex-ante type \( H \) is incentivized (the second term in (16)) to report truthfully his ex-ante type. That is, \( R_L = 0, R_H \geq 0 \).

3.3 Flexible Contract with Private Shock

The ex-ante type \( \tau \) and the shock \( \omega \), and consequently \( \theta \), are the seller’s private information and not observed by the buyer. Then, the revelation principle requires that the mechanism \( \{q^p(\tau, \omega), t^p(\tau, \omega)\} \) be incentive compatible with respect to both the ex-ante type \( \tau \) and the shock \( \omega \). Define

\[
U^p(\tau; \omega) = \max_{s(\omega)} \mathbb{E}_\omega \left\{ t^p(\tau, s(\omega)) - C(q^p(\tau, s(\omega)); \Theta(\tau, \omega)) \right\},
\]

as the seller’s optimal expected revenue with ex-ante type \( \tau \) when he reports \( \tau' \) at \( t = 1 \) and uses an arbitrary reporting strategy \( s(\omega) \) at \( t = 2 \). With some abuse of notation define \( U^p(\tau) := U^p(\tau; \omega) \). Incentive compatibility for the ex-ante type can be written as

\[
IC^p_{\tau} : \quad U^p(\tau) \geq U^p(\tau', \tau) \quad \forall \tau', \tau \in \{L, H\}. \tag{17}
\]

Define

\[
\bar{u}^p(\tau, \omega) := t^p(\tau, \omega) - C(q^p(\tau, \omega); \Theta(\tau, \omega)),
\]

as the seller’s revenue with ex-ante type \( \tau \) and shock \( \omega \) when he reports truthfully at \( t = 1 \) and \( \omega' \) at \( t = 2 \). With some abuse of notation define \( \bar{u}^p(\tau, \omega) := \bar{u}^p(\tau) \). Incentive compatibility for the shock can be written as

\[
IC^p_{\omega} : \quad \bar{u}^p(\omega) \geq \bar{u}^p(\omega', \omega) \quad \forall \omega', \omega ; \\omega \in \{\omega, \bar{\omega}\}, \forall \tau \in \{L, H\}. \tag{18}
\]

Note that the revelation principle only requires truthful reporting of the shock \( \omega \) given that the truth about \( \tau \) is told; it does not restrict any strategy \( \sigma(\omega) \) at \( t = 2 \) following a lie about \( \tau \) at \( t = 1 \).

The interim IR for the seller can be written as

\[
IR_\tau : \quad U^p(\tau) \geq 0 \quad \forall \tau \in \{L, H\}. \tag{19}
\]

The optimal forward contract design with private shock and interim IR can then be written as

\[
\max S - R \quad \text{subject to (17), (18), and (19).}
\]

**Theorem 3.** With two ex-ante types, the allocation \( q^{opt}(\tau, \omega) \) of the optimal flexible contract with private shock satisfies

\[
v(q^{opt}(L, \theta); \theta) = c(q^{opt}(L, \theta); \Theta(L, \theta)) + \frac{p_H}{p_L} \left[ \mathbb{E}_\omega \left\{ F_L(\theta) - F_L(\theta') \right\} \right], \tag{20}
\]

\[
v(q^{opt}(H, \theta); \theta) = c(q^{opt}(H, \theta); \Theta(H, \theta)); \tag{21}
\]

the associated payment function \( \bar{v}^{opt}(\tau, \theta) \) is given by

\[
\bar{v}^{opt}(\tau, \theta) = C(q^{opt}(\tau, \theta); \Theta(\tau, \theta)) + \int_0^\pi \bar{C}(q^{opt}(\tau, \theta); \Theta(\tau, \theta)) d\theta, \tag{22}
\]

\[
\bar{v}^{opt}(H, \theta) = C(q^{opt}(H, \theta); \Theta(H, \theta)) + \int_0^\pi \bar{C}(q^{opt}(H, \theta); \Theta(H, \theta)) d\theta \tag{23}
\]

Similar to the flexible contract with public shock, the selection for the ex-ante type \( H \) is efficient, while the allocation for the ex-ante type \( L \) is distorted from the efficient allocation \( q^{opt} \) to reduce the information rent for the ex-ante type \( H \). However, with private shock both ex-ante types need to be incentivized to truthfully report their private shock (the last term in bracket in (23) and (22)). These incentive terms in (23) and (22) have zero (ex-ante) expectation, therefore, do not affect the seller’s report about his ex-ante type at
is binding for ex-ante type to report truthfully his ex-ante type, while the IR constraint is binding for ex-ante type $L$. That is, $R_L = 0, R_H \geq 0$.

4. FIRM VS. FLEXIBLE CONTRACT

The flexible contract schemes investigated in Sections 3.2 and 3.3 can be interpreted as follows. At $t = 1$ the buyer proposes a menu of contracts (payment vs. quantity curves) to the seller, as in Fig. 1, where each contract curve is parametrized by shock $\omega$. The seller chooses one contract curve based on his private ex-ante type $\tau$. Then, at $t = 2$ a point from the chosen flexible contract with private (resp. public) shock is selected based on the reported (resp. observed) shock $\omega$, and the associated allocation and payment are implemented. We note that the firm contract (with penalty) can also be interpreted in a similar way. The corresponding contract menu for the firm contract scheme consists of linear contract curves with a fixed slope given by the penalty rate $\lambda$ (Fig. 1). Therefore, one can see the rationale behind the idea of the flexible contract schemes. With flexible schemes, one can expand the space of mechanism’s allocation function, from $q(\tau)$ to $q(\tau, \omega)$, and therefore, can potentially improve the resulting social welfare by refining the allocation function through inclusion of $\omega$. However, there is a trade-off as such a refinement of allocation function comes at a cost. The seller needs to be further incentivized to truthfully reveal its private shock $\omega$ in addition to truthful report of $\tau$. We show that the buyer would benefit from such a flexibility and the buyer’s utility under the flexible contracts is greater than the one under firm contract. Moreover, if the monitoring of $\omega$ is possible, the buyer would benefit from such monitoring.

**Theorem 4.** Among the three different optimal contract schemes considered above, the buyer’s total utility is the highest from the forward contract under uncertainty with public shock and is the lowest from the firm forward contract. That is, $S_1 - R_1 \geq S'_1 - R'_1 \geq S''_1 - R''_1$.\(^2\)

We would like to emphasize that theorem 4 does not imply any ordering for the resulting information rent $R$ or the resulting social welfare $S$ under different schemes. It only states that the aggregate effect of the changes in the social welfare $S$ and the information rent $R$ under different schemes results in the order on the buyer’s utility $S - R$ described by Theorem 4.\(^3\)

When dealing with energy resources with different uncertainty levels (the MPS condition), the buyer can differentiate between different types of the seller by offering flexible contracts instead of firm contracts. As a result, the buyer can infer about the uncertainty of the seller’s resources by observing the seller’s choice in forward markets. With flexible contracts, the seller with more uncertain resources (type $H$) receives a higher utility.

\(^2\)The result holds for any finite number of ex-ante type $\tau$.

\(^3\)See [13] for an example where the social welfare is higher under private shock than the the one under public shock.

We note that the imposed IR constraint for all the three contracts is interim, that is, each ex-ante type gets a positive expected payoff, $R_\tau \geq 0$. Therefore, it is possible that for some realizations of $\omega$ ex-post IR is violated, i.e. $t(\tau, \omega) < C(q(\tau, \omega); \Theta(\tau, \omega))$. In fact, for a sequential contract with private shock, ex-post IR is definitely violated for some realizations of $\omega$. Therefore, the buyer’s utility would be reduced if one replaces interim IR with ex-post IR. With interim IR the buyer can penalize the seller for some realizations of $\omega$ (resulting in negative utility for the seller), and consequently, leaves him less information rent than in the case with ex-post IR. Characterizing sequential contracts with ex-post IR (no penalty) and generalizing the results presented in this paper to the case with many sellers are the research topics that we are currently investigating.

5. REFERENCES


