Two-Stage Game Theoretic Modelling of Airline Frequency and Fare Competition

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ABSTRACT

Airlines make capacity and fare decisions in a competitive environment. Capacity decisions, encompassing decisions about frequency of service and seats-per-flight, affect both the operating costs and revenues of airlines. These decisions have significant implications for the performance of the air transportation system as a whole. Capacity and fare decisions of different airlines are interdependent, both serving as tools in an airlines competitive arsenal. This interdependency motivates a game theoretic approach to modeling the decision process. Capacity (especially frequency) decisions are typically made months in advance of flight departure, with only an approximate knowledge of what fares will be, while fare decisions are made weeks to minutes ahead of flight departure. Several studies have stressed the need to develop two-stage game theoretic models to account for the sequential nature of these decisions, but there are very few analytical, computational, or empirical results available for such models. In this article (working paper available at link in [1]), we develop a two-stage frequency and fare competition model, demonstrate its tractability across a wide range of assumptions, and validate its predictions against observed airline behavior.

We take the payoff function of an airline operating in a set of markets to be the sum of the differences between revenues and costs in those markets, with costs as a linear function of flight frequency. To compute revenue, we explore two commonly used multinomial logit models of market share. Frequency decisions are made in the first stage of the game, while fare decisions are made in the second stage. We begin our analysis with a simplified version of this game: two airlines competing in a single market, with no connecting passengers, infinite seating capacity, and the absence of a no-fly alternative. Under these assumptions, for either market share model, we are able to prove that (1) the second-stage fare game always has unique pure strategy Nash equilibrium (PSNE), (2) first-stage payoffs for each airline are concave with respect to that airline's frequency strategy across plausible utility parameter ranges, and (3) first-stage payoffs for each airline are submodular functions in the overall frequency strategy space. As the game is two-player, (3) means that by changing the sign of one player's strategy space, we can trivially convert the game into a supermodular game. These results demonstrate that subgame-perfect PSNE is a credible and tractable solution concept for our model. In particular, the existence and uniqueness results indicate the suitability of PSNE as a solution concept for the second-stage game. Concave payoffs ensure that individual first-stage payoff maximization problems are efficiently solvable, and supermodularity ensures that several iterative learning dynamics converge to equilibrium [2]. We then relax each of the assumptions made in this simplified model by computationally solving the second stage fare game, generating equilibrium fare decisions and profits for every set of frequency decisions for integer daily frequency values ranging from 1 to 20, for various numbers of players, seats per flight, and values of utility parameters (including the no-fly option). Then, we fit quadratic approximations to these profits as functions of the frequencies of all players. We find an excellent fit ($R^2 > 0.9$) in all cases. Additionally, the signs of all estimated coefficients are consistent with submodularity and concavity properties demonstrated earlier. We show that for an N-player game with such concave and submodular quadratic payoff functions, the myopic best response heuristic, where each player optimizes its payoff against fixed opponent strategies iteratively, converges to a PSNE.

To test the tractability and predictive validity of our model in practice, we apply it to a 4-airline, 11-airport network in the Western U.S, using publicly available airline operations data. We use the quadratic functions of airline frequency fitted above and additionally enforce the aircraft availability constraints, and solve for equilibrium iteratively using the myopic best response heuristic. To calibrate the 11 free quadratic payoff coefficients of this model, we use a stochastic gradient approximation algorithm to minimize the absolute errors between observed and predicted frequency strategies. In practice, the game convergences to equilibrium quickly, and the calibrated model's frequency predictions approximate observed behavior both in-sample and out-of-sample, suggesting that refinements of the model could be pursued for use in scenario analysis, forecasting, planning, and policy making.

1. REFERENCES

[1] R. Harder and V. Vaze. Two stage game theoretic modelling of airline frequency and fare competition. *Working Paper*, `http://engineering.dartmouth.edu/mdsg/harder_vaze_2stage.pdf`, 2016.

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